International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN (P): 2319–3972; ISSN (E): 2319–3980
Vol. 11, Issue 1, Jan–Jun 2022; 25–32

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

SOME RESULTS ON INTUITIONISTIC FUZZY METRIC SPACE

Dr. Preeti Sengar

Research Scholar, SGSITS Indore, Madhya Pradesh, India

ABSTRACT

In this paper, we establish some common fixed point theorems we simplify the results of Chouhan and Kumar [4] from FMS to IFMS in IFMS for series of self-mappings making use of an implicit relation and the typical property (E.A) in which.

KEYWORDS: *FMS, IFMS, Semi-Compatible Sub-Sequentially Continuous Mappings*

Article History

Received: *03 Apr 2022* | ***Revised:*** *07 Apr 2022* | ***Accepted:*** *07 May 2022*

1. INTRODUCTION

Atanassov generalized the idea of fuzzy set by launching the conception of intutionistic fuzzy set and thereafter numerous authors (Manro et al. 2012; Alaca et al. 2006; Park 2004; Turkoglu et al. 2006) did contribution that is remarkable the field of intutionistic fuzzy sets. Al-Thagafi and Shahzad (2008) defined sporadically weakly compatible mappings in IFMS which is more basic than the idea of weakly mappings that are appropriate. They indicated that occasionally mappings which can be weakly appropriate weakly suitable mappings but converse is perhaps not fundamentally real.

DEFINITION

2.1. A operation that is binary : [0, 1] × [0, 1] → [0, 1] is called a continuous t-norm if ∧ satisfies the following axioms:

i. Is continuous, commutative , associative;

ii. 1 = ı p belongs to closed interval 0,1;

iii. m ≤ n o whenever ı ≤ n , m ≤ o p, q, r, s belongs to closed interval 0,1.

Examples of t-norm are l m = min {l,m} and lm=lm.

2.2. A operation that is binary : [0, 1] × [0, 1] → [0, 1] is called continuous t-conorm if ρ satisfies the following axioms:

i. ρ is continuous, commutative, associative;

ii. d ρ 0 = d d ∈ [0, 1];

iii. d ρ e ≤ f ρ g whenever d ≤ f , e ≤ g a, b, c, d belongs to closed interval 0,1.

Examples of t-norm are d e=min {d, e}, d ρ e=de.

2.3. A tuple (Y, U, V, , ρ) is called an IFMS if Y is an any set, and ρ is a continuous t-norm and t-co-norm and U,V are fuzzy sets on an interval Y^2 × [0,∞) satisfying following axioms:

(i) $U(m, n, t) + V(m, n, t) \leq 1$ m, n ∈ Y and t > 0;

(ii) $U(m, n, 0) = 0$ m, n ∈ Y;

(iii) $U(m, n, t) = 1$ m, n ∈ Y and t > 0 if and only if m = n;

(iv) $U(m, n, t) = U(n, m, t)$ m, n ∈ Y and t > 0;

(v) $U(m, n, t) * U(n, o, s) \leq U(m, o, t + s)$ m, n, o ∈ Y and s, t > 0;

(vi) $U(m, n, \cdot) : [0,\infty) \to [0, 1]$ is left continuous, m, n ∈ Y, ;

(vii) $\lim_{n\to\infty} U(m, n, t) = 1$ m, n ∈ Y and t > 0;

(viii) $V(m, n, 0) = 1$ m, n ∈ Y;

(ix) m = n iff $V(m, n, t) = 0$ m, n ∈ Y and t > 0;

(x) $V(m, n, t) = V(n, m, t)$ m, n ∈ Y and t > 0;

(xi) $V(m, n, t)\ \rho\ V(n, o, s) \geq V(m, o, t + s)$ m, n, o ∈ Y and s, t > 0;

(xii) $V(m, n, \cdot) : [0,\infty) \to [0, 1]$ is right continuous m, n ∈ Y;

(xiii) $\lim_{n\to\infty} V(m, n, t) = 0$ m, n ∈ Y.

2.6. Let (Y, U, V, *, ◊) be an FMS. Then a sequence $\{x_n\}$ in X is called

(i) Convergent to a point m ∈ Y if

$\lim_{n\to\infty} U(x_n, m, h) = 1$, $\lim_{n\to\infty} V(x_n, m, h) = 0$ h > 0,

(ii) Cauchy sequence if

$\lim_{n\to\infty} U(x_{n+q}, x_n, h) = 1$, $\lim_{n\to\infty} V(x_{n+q}, x_n, h) = 0$ ∀ t > 0 and q > 0.

2.7. An FMS (Y, U, V,*, ◊) s called complete if and only if every Cauchy sequence in Y is convergent.

2.8. Let P, Q be self-mappings of an FMS (Y, U, V,*, ρ) Then a pair (P, Q) called commuting if $U(PQx, QPx, t) = 1$, $V(PQx, QPx, t) = 0$.

2.9 A pair of self-maps (k, l) of an FMS (Y, U, V, ∧, ρ) is said to be compatible if $\lim_{n\to\infty} U(klx_n, lkx_n, h) = 1$, $\lim_{n\to\infty} V(klx_n, lkx_n, h) = 0$ for every h > 0, whenever $\{x_n\}$ is a sequence x_n in Y : $\lim_{n\to\infty} kx_n = \lim_{n\to\infty} lx_n = y$ for some yY.

2.10. Allow P and Q be self-mappings of an IFMS (X,U,V,*, ρ).Then a pair (P, Q) is called Sub-compatible if $\lim_{(n\to\infty)} U(PQxn, QPxn, h) = 1$, $\lim_{(n\to\infty)} V(PQxn, QPxn, h) = 0$ for all h > 0, whenever {xn} is a sequence in Y : $\lim_{(n\to\infty)} Pxn = \lim_{(n\to\infty)} Qxn = u$ for some u ∈ Y.

3. Main Result

Theorem 2.1. Let R, S, C, D, E and F be self-mappings of IIFMS (Y, U, V,∧, ρ) well-defined by h*h≥t , (1-h) ρ (1-h)≤(1-h) ∀ a, b∈[0,1].If the pairs (R,EF) , (S, CD) are semi-compatible , sub-sequentially constant mappings, then

- The set (R, EF),(S, CD) are semi-compatible , sub-sequentially constant mappings.
- Further, the mappings R, S, C, D, E and F take a single common fixed point in Y providing the complex maps equation that is fulfill

  U^2(Rx,Sy,h)*[U(EFx,Rx,h)*U(CDy,Sy,h)]

  [pU (EFx,Rx,h)+qU(EFx,CDy,h)]U(EFx,Ly,h)

  and V2(Rx,Sy,h) ρ [V(EFx,Rx,h)◊V(CDy,Sy,h)]

  [pV(EFx,Rx,h)+qV(EFx,CDy,h)]V(EFx,Ly,h) (1.1)

  i x, y Y , h > 0, where 0<p,q<1 , p+q=1.

Proof:

We realize that pairs (R, EF) and (S, CD) are Semi-compatible and Sub-sequentially continuous mappings, there ∃ a sequence {xn} in Y

$\lim_{n\to\infty} Rx_n = \lim_{n\to\infty} EFx_n = p$ for some h Y

and $\lim_{n\to\infty} U(R(EF)x_n, (EF)Rx_n, t) =1$, h<0

and $\lim_{n\to\infty} U(Rp, EFp, t)=1$ (1.2)

then we have Rp= EFp,

Similarly $\lim_{n\to\infty} Sy_n = \lim_{n\to\infty} CDy_n = g$ Y

$\lim_{n\to\infty} U(S(CD)y_n, (CD)Sy_n, t) =1$, i h<0

and $\lim_{n\to\infty} U(Sg,CDg,t)=1$ **(1.3)**

Henceforth t and g is a coincidence point of (R, EF),(S, CD).

then we become Rp=EFp and Sg=CDg. **(1.4)**

Step 1:- first we prove p = g. Put $x = x_n$, $y = y_n$ in equation (1.1)

$U^2(Rx_n,Sy_n,h)[U(EFx_n,Rx_n,h).U(CDy_n,Sy_n,h)]$

≥[pU(EFxn,Rxn,h)+qU(EFxn,CDyn,h)]U(EFxn,Syn,h)

and V2(Rxn,Syn,h) ρ [V(EFxn,Lxn,h) ρ V(CDyn,Syn,h)]

[pV(EFxn,Rxn,h)+qV(EFxn,CDyn,h)]V(EFxn,Syn,h)

Now, U2(p,g,h)   [U(p,p,h).U(g,g,h)] i i pU[(p,p,h)+qU(p,g,h)]U(p,g,h)

and V2(p,g,h) ρ [V(p,p,h) ρ V(g,g,h)] i pV[(p,p,h)+qV(p,g,h)]V(p,g,h)

U2(p,g,h)≥[p+qU(p,g,h)]U(p,g,h)

and V2(p,g,h)≤[p+qV(p,g,h)]V(p,g,h)

U(p,g,h)p1-q and V (p,g,h)p1-q

U (p,g,h) =1 and V(p,g,h)=0. **(1.6)**

Thus we have p = g

Step 2:-again we prove that Rp = p, Put x=p , y=y_n in equation (1.1)

$U^2(Rz,Sy_n,h)[U(EFz,Rz,h).U(CDy_n,Sy_n,h)]$ i i $[pU(EFz,Rz,h)+qU(EFz,CDy_n,h)]U(EFz,Sy_n, h)$

and

$V^2(Rz,Sy_n,h)\rho[V(EFz,Rz,h)\rho V(CDy_n,Sy_n,h)]$ i $[pN(EFz,Rz,h)+qV(EFz,CDy_n,h)]VEFz, Sy_n, h)$

$U^2(Rp,g,h)_\wedge[U(EFp,Rp,h).U(g,g,h)]$ [pU(EFp,Rp,h)+qU(EFp,g,h)]U(EFp, g, h)

and

V2(Rp, g, h) ρ [V(EFp,Rp,h) ρ V(g,g,h)][pV(EFp,Rp,h)+qV(EFp,g,h)]V(EFp, g, h)

U2(Rp,g,h) [U(Rp,Rp, h).U(g,g,h)] [pU(Rp,Rp,h)+qU(Rp,g,h)]U(Rp, g, h)

and V2(Rp,g,h) ρ [V(Rp,Rp,h) ρ V(g,g,h)] [pV(Rp,Rp,h)+qV(Rp,g,h)]V(Rp, g, h

U2(Rp,g,h) [p+qU(Rp,g,h)]U(Rp,g,h)

and V2(Rp,g,h) [p+qV(Rp,g,h)]V(Rp,g,h)

U(Rp,g,h) p1-q and V(Rp,g,h) p1-q

U(Rp,g, h) =1 and V(Rp,g,h)=0

Hence Rp = g = p.

Step 3:- In this step we prove Cp = p

Then we use x=xn, y=p n (1.1)

U2(Rxn,Sp,h)[U(EFxn,Rxn,h)U(CDp,Sp,h)]

[pU(EFxn,Rxn,h)+qU(EFxn,CDp, h)] U(EFxn,Sp, h)

and V2(Rxn,Sp,h) ρ [V(EFxn,Rxn,h) ρ V(CDp,Sp,h)]

[pV(EFxn,Rxn,h)+qV(EFxn,CDp, h)] V(EFxn,Sph)

U2(p,Sp,h) [U(Sp,Sp,h).U(p,p,h)]

[pU(Sp, Sp,h)+qU(p, Sp, h)]U(p,Sp, h)

and V2(p,Sp,h) ρ [V(Sp,Sp,h) ρ V(p,p,h)]

[pV(Sp,Sp,h)+qV(p,Sp,h)]V(p, Sp, h)

U2(p,Sp,h) [p+qU(p,Sp,h)] U(p,Sp,h)

and V2(p,Sp,h) [p+qV(p,Sp,h)] V(p,Sp,h)

U(p,Sp,h) p1-q and V(p,Sp,h) p1-q

U(p,Sp,h) =1 , V(p,Sp,h)=0

we get p=Sp **(1.8)**

Step 4:-Again we claim that Sp=p,

Put x=Dp , y=p in (1.1)

U^2(RDp,Sp,h)[U(EF(Dp),Rp,h).U(CDp,Sp,h)]

[pU(EF(Dp),Rp,h)+qU(EF(Dp),CDp, h)] U (EF (Dp), Sp, h)

and V^2(RDz,Sp,h) ρ [V(EF(Dp),Rp,h) ρ V(CDp,Sp,h)]

[pV(EF(Dp),Rp,h)+qV(EF(Dp),CDp,h)] V (EF (Dp), Sp,h)

U^2(RDp,p,h)[U(EF(Dp),Rp,h).U(CDp,Sp,h)]

[pU(L(Dp),Lp,h)+qU(L(Dp),Sp, h)] U(L(Dp), Cp, h)

and V^2(RDp,p,h) ρ [V(EF(Dp),Rp,h) ρ V(CDp,Sp,h)]

[pV(L(Dp),Lp,h)+qV(L(Dp),Sp, h)] V(L(Dp), Cp, h)

U^2(Dp,p,h)∧[U(Dp,Dp,h).U(p,p,h)]

[pU(Dp,Dz,h)+qU(Dp,p,h)] U(Dp,p,h)

U^2(Dp,p,h) [p+qU(Dp,p,h)] U(Dp,p,h)

and V^2(Dp,p,h) ρ[p+qV(Dp,p,h)] V(Dp,p,h)

U(Dp,p,h) p1-q and V(Dp,p,h) p1-q

U(Dp,p,h) =1 and V(Dp,p,h)=0

We get Dp = p **(1.9)**

Step 5:- Once again we show that Cp=p,

Put x=Cp, y=p in (1.1)

U2(ACp,Bp,h)[U(EFCp,Ap,h).U(CDp,Bp,h)]

[pU(EFCp, ACp, h)+qU(EFCp, CDp, h)] U(EFCp,Bp,h)

and V2(ACp,Bp,h) ρ [V(EFCp,Ap,h) ρ V(CDp,Bp,h)]

[pV(EFCp,ACp,h)+qV(EFCp,CDp, h)] V(EFCp,Bp,h)

U2(Cp,p,h) [U(Ap,Ap,h).U(p,p,h)] [pU(Ap,Ap,h)+qU(Cp,p, h)] U(Cp, p, h)

and

V2(Cp,p,h) ρ [V(Ap,Ap,h) ρ V(p,p,h)] [pV(Ap,Ap,h)+qV(Cp,p, hh)] V(Cp, p, h)

U2(Cp,p,h) [p+qU(Cp,p,ht)] U(Cp,p,h)

and V2(Cp,p,h) [p+qV(Cp,p,h)] V(Cp,p,h)

U(Cp,p,h) p1-q and V(Cp,p,h) p1-q

U(Cp,p,h) =1 and V(Cp,p,h)=0

We get Cp=p **(1.10)**

Step 6:- Once more we prove that Fp=p,

Put x=Fp , y=p in (1.1)

U2(AFp,p,h) [U(EF(Fp),Ap,h).U(CD(Fp),Bp,h)]

[pU(EF(Fp),A(Fp),h)+qU(EF(Fp),CDp, h)]U(EF(Fp),Bp, h)

and V2(AFp,p,h) ρ [V(EF(Fp),Ap,h) ρ V(CD(Fp),Bp,h)]

[pV(EF(Fp),A(Fp),h)+qV(EF(Fp),CDp, h)]V (EF(Fp),Bp,h)

U2(Fp,p,h) [U(Fp,Fp,h).U(Bp,Bp,h)]

[pU(Fp,Fp,h)+qU(Fp,p, h)] U(Fp, p, h)

and V2(Fp,p,h) ρ [V(Fp,Fp,h) ρ V(Bp,Bp,h)]

[pV(Fp,Fp,h)+qV(Fp,p,h)]V(Fp,p,h)

U2(Fp,p,h) [p+qU(Fp,p,h)] U(Fp,p,h)

and V2(Fp,p,h) ρ[p+qV(Fp,p,h)] V(Fp,p,h)

U(Fp,p,h) p1-q and V(Fp,p,h) p1-q

U(Fp,p,h) =1 and V(Fp,p,h)=0

We get Fp=p **(1.11)**

Step 7:- Once more we prove that Ep=p, put y=p and x=Ep in (1.1) U^2(AEp,Bp,h)$_\wedge$[U(EEp,Ap,h).U(CDp,Bp,h)]

[pU(EFEp,Ap,h)+qU(EFEp,CDp, h)] U(EFEp, Bp, h)

and V2(AEp,Bp,h) ρ [V(EEp,Ap,h).V(CDp,Bp,h)]

[pV(EFEp,Ap,h)+qV(EFEp,CDp, h)] V(EFEp, Bp, h)

U2(Ep,p,h) [U(Ep,Ep,h).U(p,p,h) [pU(Ep,Ep,h)+qU(Ep,p, h)] U(Ep, p, h)

and V2(Ep,p,h) ρ [V(Ep,Ep,h) ρ V(p,p,h)] [pV(Ep,Ep,h)+qV(Ep,p, h)] V(Ep, p, h)

U2(Ep,p,h) [p+qU(Ep,p,h)] U(Ep,p,h)

and V2(Ep,p,h) [p+qV(Ep,p,h)] V(Ep,p,h)

U(Ep,p,h) p1-q and V(Ep,p,h) p1-q

U(Ep,p,h) =1 and V(Ep,p,h)=0

We get Ep=p **(1.12)**

i.e. Rp=Sp=Cp=Dp=Ep=Fp=p

Ergo p is a common point that is fixed of S, C, D,E , F.

Uniqueness: - Let s be another common point that is fixed of S, C, D, E , F. Suppose t≠s

Put x=p, y= s in (1.1)

U2(Rp,Ss,h) [U(EFp,Rp,h).U(CDs,Ss,h)][pU(EFp,Rp,h)+qU(EFp,CDs,h)]U(EFp,Ss,h)

and V2(Rp,Ss,h) ρ [V(EFp,Rp,h).V(CDs,Ss,h)] [pV(EFp,Rp,h)+qV(EFp,CDs,h)]V(EFp,Ss,h)

U2(Rp,Ss,h) [U(Rp,Rp,h).U(Ss,Ss,h)] i [pU(Rp,Rp,h)+qU(Rp,Ss,h)]U(Rp,Ss,h)

and V2(Rp,Ss,h) ρ [V(Rp,Rp,h).V(Ss,Ss,h)] i [pV(Rp,Rp,h)+qV(Rp,Ss,h)]U(Rp,Sp,h)

U2(p,s,h) i [p+qU(p,s,h)]U(p,s,h)

and V2(p,s,h) [p+qV(p,s,h)]V(p,s,h)

U(p,s,h) i i p1-q and V(p,s,h) p1-q

U (p,s,h) =1 and V(p,s,h)=0 We get p=s.

Corollary

Allow P, Q, R and S be self-maps of FMS (Y, U, V,∧, ρ) with constant t-co-norm and t-norm that is constant defined by t∧t ≥ t and (1-t) ρ (1-t) ≤ (1-t) ∀ a, b ∈[0,1].If the pairs (P,S) and (Q,R) are often Weak-compatible then

- The set (P, S) ,(Q, R) has a coincidence point.
- Further, the mapping P, Q,R and S have actually a unique point that is common is fixed X supplied the involved maps meet the inequality

U2(Px,Qy,t) [U(Sx,Px,t)*U(Ry,Qy,t)] [pU(Sx,Px,t)+qU(Sx,Ry,t)]U(Sx,Qy,t)

and V2(Px,Qy,t) ρ [V(Sx,Px,t) ρ V(Ry,Qy,t)]

[pV(Sx,Px,t)+qV(Sx,Ry,t)]V(Sx,Qy,t)

x, y Y and t > 0, where 0<p,q<1 and p+q=1.

CONCLUSION

The results improve and extent the scope of the study of common point that is fixed from the course of semi-compatible mappings to a wider class of sub-sequentially continuous mapping in IFMS.

REFERENCES

1. *Alaca C., Turkoglu D. and Yildiz C., Fixed points n FMSs, Smallerit Choas, Solitons & Fractals, 29(5)(2006), 1073-1078.*
2. *Atanassov K., Intuitionistic Fuzzy sets, Fuzzy sets and system, 20(1986) 87- 96.*

3. *Banach S., "Theorie Les Operations Lineaires, Manograie Mathematyezne Warsaw Poland," n French, Z Subwencji Funduszu Kultury Narodowej, New York, 1932.*
4. *Chauhan S., and Kumar S., Common fixed point theorems for compatible and subsequentially continuous mappings n fuzzy metric space, Kragujevac J. Math. 36(2) (2012), pp.225-235.*
5. *Coker D., "An Introduction to Intuitionistic Fuzzy Topological Spaces," Fuzzy Sets and System, Vol. 88, No. 1, 1997, pp. 81-89.*
6. *Gregori V., Romaguera S., and Veeramani P., "A Note on FMSs," Chaos, Solitons and Fractals, Vol. 28, No. 4, 2006, pp. 902-905.*
7. *Jungck G., Commuting mappings and fixed points, Amer. Math. Monthly, 83(1976), 261-263.*
8. *Kramosil O., and Michalek J., Fuzzy metric and statistical metric spaces, Kybernetika, 11(1975).*
9. *Pant R. P., "Common Fixed Points of Noncommuting Mappings," Journal of Mathematical Analysis and Applications, Vol. 188, No. 2, 1994, pp. 436-440.*
10. *Park J. H., FMSs, chaos, Solitions & Fractals 22(2004), 1039- 1046.*
11. *Saadati R., and Park J. H., "On the Intuitionistic Fuzzy Topological Spaces," Chaos, Solitons and Fractals, Vol. 27, No. 2, 2006, pp. 331-344.*
12. *Schweizer B., and Sklar A., "Statistical Metric Spaces," Pacific Journal Mathematic, Vol. 10, 1960, pp. 314-334.*
13. *Turkoglu D., Alaca C., Cho Y.J., and Yildiz C., "Common Fixed Point Theorems n FMSs," Journal of Applied Mathematics and Computing, Vol. 22, No. 1-2, 2006, pp. 411-424.*
14. *Zadeh L.A., Fuzzy sets, Inform. and Control 8(1965), 338-353.*